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Muon Capture in Hydrogen.

A. FUJII
Laboratori Nazionali di Frascati del C.N.E.N. - Frascati (Roma)

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In this note we estimate the rate of muon capture by proton [1,2]

$$\mu^- + p \to n + \nu\,, \tag{1}$$

to the best of our present knowledge within the framework of the universal $V—A$ theory and compare it with the experimental capture rate in liquid hydrogen [3-5].

The matrix element of the reaction (1) is written as

$$\langle n\nu|H_{\text{int}}|p\mu\rangle = \frac{G}{\sqrt{2}}\cdot\frac{-i}{(2\pi)^2}\cdot\delta(n+\nu-p-\mu)\sqrt{\frac{m_n}{\varepsilon_n}}\sqrt{\frac{m_p}{\varepsilon_p}}\sqrt{\frac{m_\mu}{\varepsilon_\mu}}\sqrt{\frac{m_\nu}{\varepsilon_\nu}}\cdot \cdot\overline{u}(n)(V_\lambda+P_\lambda)u(p)\cdot\overline{u}(\nu)\gamma_\lambda(1+\gamma_5)u(\mu)\,, \tag{2}$$

where G is the Fermi coupling constant, V_λ and P_λ are the weak nucleon-nucleon vector and axial vector current respectively, and the particle names also stand for their four-momenta. Let us assume Lorentz invariance and definite G-conjugation parity for the weak currents, *viz.*

$$GV_\lambda G^{-1} = V_\lambda\,, \qquad GP_\lambda G^{-1} = -P_\lambda\,, \tag{3}$$

then the currents are written in a general form

$$V_\lambda + P_\lambda = f_1\gamma_\lambda - \frac{if_2}{m}\sigma_{\lambda\varrho}q_\varrho + g_1\gamma_\lambda\gamma_5 - \frac{ig_2}{m}q_\lambda\gamma_5\,, \tag{4}$$

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(1) H. PRIMAKOFF: *Rev. Mod. Phys.*, **31**, 802 (1959).
(2) IA. B. ZEL'DOVICH and S. S. GERSHTEIN: *Sov. Phys. JETP*, **8**, 570 (1959).
(3) R. H. HILDEBRAND: *Phys. Rev. Lett.*, **8**, 34 (1962).
(4) E. BLESER, L. LEDERMAN, J. ROSEN, J. ROTHBERG and E. ZAVATTINI: *Phys. Rev. Lett.*, **8**, 288 (1962).
(5) E. BERTOLINI, A. CITRON, G. GIALANELLA, S. FOCARDI, A. MUKHIN, C. RUBBIA and S. SAPORETTI: *Proc. of 1962 Intern. Conf. on High-Energy Physics* (CERN, 1962), p. 421.

where

$$m = m_p = m_n\,, \qquad q = n - p\,, \qquad \sigma_{\lambda\varrho} = \frac{1}{2i}(\gamma_\lambda\gamma_\varrho - \gamma_\varrho\gamma_\lambda)\,.$$

The form factors f_1, f_2, g_1, g_2 are dimensionless functions of $z=q^2/m^2$ known as the vector, weak magnetism, axial vector and induced pseudoscalar form factors respectively, and normalized such that $f_1(0)=1$. They can be chosen all real if time reversal invariance is imposed. The capture rate of the reaction (1) depends critically on the relative spin orientation of the proton and the muon (6). The capture rate in the 1s-orbital, spin triplet ($F=1$) mesic atom state w_1 and that in the 1s-orbital, spin singlet ($F=0$) mesic atom state w_0 are given by Adams' formula (7)

$$(5)\quad w_1 = 240\ \text{s}^{-1}\cdot[0.132|f_1|^2 + 1.97\cdot10^{-3}|f_2|^2 + 0.132|g_1|^2 + 4.47\cdot10^{-6}|g_2|^2 - 1.84\cdot10^{-2}\,\text{Re}(f_1f_2^*) - 0.264\,\text{Re}(f_1g_1^*) + 5.83\cdot10^{-4}\,\text{Re}(f_1g_2^*) + 1.84\cdot10^{-2}\,\text{Re}(f_2g_1^*) + 1.01\cdot10^{-4}\,\text{Re}(f_2g_2^*) - 5.83\cdot10^{-4}\,\text{Re}(g_1g_2^*)]\,,$$

$$(6)\quad w_0 = 240\ \text{s}^{-1}\cdot[0.171|f_1|^2 + 5.34\cdot10^{-3}|f_2|^2 + 1.17|g_1|^2 + 4.45\cdot10^{-6}|g_2|^2 + 6.04\cdot10^{-2}\,\text{Re}(f_1f_2^*) + 0.892\,\text{Re}(f_1g_1^*) - 1.74\cdot10^{-3}\,\text{Re}(f_1g_2^*) + 0.158\,\text{Re}(f_2g_1^*) - 3.08\cdot10^{-4}\,\text{Re}(f_2g_2^*) - 4.56\cdot10^{-3}\,\text{Re}(g_1g_2^*)]\,.$$

In liquid hydrogen the capture takes place in the muonic ortho-hydrogen molecular ion (8) and the experimentally observed molecular capture rate w is related to the atomic hyperfine capture rate w_1, w_0 by

$$(7)\qquad w = 2\gamma(\eta w_0 + (1-\eta)w_1)\,.$$

The factor γ is the measure of the overlap of the muon and the proton, namely the ratio of the muon density at the position of either proton in the ortho-molecular ion and the muon density at the position of the proton in the muonic atom. The parameter η depends on the strengths of the spin-orbit and spin-spin couplings in the molecular ion, which mix the states of the total spin $\frac{3}{2}$ with those of the total spin $\frac{1}{2}$. WEINBERG discussed these molecular parameters in detail (8) and gave the estimate $2\gamma=1.17$ and $\eta=\frac{3}{4}$. Equation (7) then reads

$$(8)\qquad w = 1.17\cdot(\tfrac{3}{4}w_0 + \tfrac{1}{4}w_1)\,.$$

The hypothesis of the conserved vector current (9) (c.v.c.) requires that the functional forms of the vector and weak magnetism form factors are equal to those

(6) J. BERNSTEIN, T. D. LEE, C. N. YANG and H. PRIMAKOFF: *Phys. Rev.*, **111**, 313 (1958).
(7) J. B. ADAMS: *Phys. Rev.*, **126**, 1567 (1962).
(8) S. WEINBERG: *Phys. Rev. Lett.*, **4**, 575 (1960).
(9) M. GELL-MANN: *Phys. Rev.*, **111**, 362 (1958).

of the electromagnetic form factors of the nucleon, which are known empirically [10]

(9) $$f_1(z) = -0.20 + \frac{1.20}{1 + 2.268z},$$

(10) $$f_2(z) = \frac{\mu_p - \mu_n}{2} \cdot f_1(z) = 1.853 f_1(z)\,.$$

The muon capture takes place at $z = 0.0114$, thus

(11) $$f_1(z) = 0.9698\,,$$

(12) $$f_2(z) = 1.797\,.$$

Nothing is known experimentally for the axial vector and the induced pseudoscalar form factors, but they can be approximately expressed by a single parameter. The axial vector form factor may well be approximated by a linear function in the vicinity of $z = 0$

(13) $$g_1(z) = g_1(0)\cdot(1 - \tfrac{1}{6}\varrho_A \cdot z + \ldots)\,,$$

where ϱ_A defined by this expansion is a dimensionless parameter, which we shall call the «A-radius» of the nucleon. The «V-radius» can be similarly defined by the expansion

$$f_1(z) = f_1(0)\cdot(1 - \tfrac{1}{6}\varrho_V \cdot z + \ldots)\,,$$

and is related to the electromagnetic mean square radius of the nucleon $\sqrt{\langle r^2 \rangle}$ by

$$\varrho_V = m^2 \cdot \langle r^2 \rangle\,.$$

The empirical form factor (9) gives $\varrho_V = 16.33$ and $\sqrt{\langle r^2 \rangle} = 0.85$ fermi. The hypothesis of the asymptotically conserved axial vector current [11,12] (a.c.a.c.) leads to the equation

(14) $$2m g_1(q^2) - \frac{q^2}{m}\, g_2(q^2) = \frac{a_\pi g_\pi}{q^2 + m_\pi^2} + m\,\varphi(q^2)\,,$$

where a_π is the pion decay amplitude, in terms of which the pion decay rate becomes

$$w(\pi^+ \to \mu^+ + \nu) = \frac{G^2}{16\pi}\left(\frac{m_\mu}{m_\pi}\right)^2 \left(1 - \left(\frac{m_\mu}{m_\pi}\right)^2\right)\cdot a_\pi^2\,,$$

g_π is the pion-nucleon coupling constant, and $\varphi(q^2)$ is a function regular in the vicinity of $q^2 = -m_\pi^2$ with the property $\varphi(-m_\pi^2) = 0$. The function $\varphi(q^2)$ represents

(10) R. Hofstadter and R. Herman: *Phys. Rev. Lett.*, **6**, 293 (1961).
(11) Y. Nambu: *Phys. Rev. Lett.*, **4**, 380 (1960).
(12) J. Bernstein, S. Fubini, M. Gell-Mann and W. Thirring: *Nuovo Cimento*, **17**, 757 (1960).

all higher order contributions to the pion propagator, and may well be regarded essentially as a constant in the small region of $|q^2| \lesssim m_\pi^2$. We obtain the Goldberger-Treiman relation ([13]) from eq. (14) by putting $q^2 = 0$,

$$2mg_1(0) = \frac{a_\pi g_\pi}{m_\pi^2} + m\,\varphi(0)\,,$$

and rewrite eq. (14) as

$$g_2(q^2) = \frac{a_\pi g_\pi}{mm_\pi^2}\cdot\frac{m^2}{q^2+m_\pi^2} + \frac{2m^2}{q^2}\cdot[g_1(q^2) - g_1(0)]\,,$$

or

$$g_2(z) = \frac{a_\pi g_\pi}{mm_\pi^2}\cdot\frac{1}{z+0.0221} - \frac{1}{3}\varrho_A\cdot g_1(0)\,.$$

From the experimental pion decay rate we obtain

$$\frac{a_\pi g_\pi}{mm_\pi^2} = \pm\,2.70 \tag{16}$$

with $g_\pi^2/4\pi = 15$, hence for $z = 0.0114$

$$g_1(z) = g_1(0)\cdot(1 - 0.0019\,\varrho_A)\,, \tag{17}$$

$$g_2(z) = \pm\,64.18 - \tfrac{1}{3}\varrho_A\cdot g_1(0)\,. \tag{18}$$

We wish to emphasize that the ratio of the induced pseudoscalar and axial vector coupling constants

$$\frac{g_P}{g_A} = \left.\frac{(m_\mu/m)\cdot g_2(z)}{g_1(z)}\right|_{z=0.0114}$$

cannot exceed ~ 8 as far as the a.c.a.c. hypothesis and the choice $a_\pi > 0$ (*viz.* $|\varphi(0)| \ll 1$) are adopted.

In Table I we summarize the computed hyperfine and molecular capture rates with the form factors (11), (12), (17), (18) and eq. (8), for the choice of the parameters $a_\pi > 0$, $g_1(0) = 1.25$ and 1.20, and $\varrho_A = 0$, ϱ_V, $2\varrho_V$ and $3\varrho_V$. For the purpose of reference we also included the original Fujii-Primakoff's choice ([14])

$$\left\{\begin{aligned} f_1(z) &= 0.972\\ f_2(z) &= 1.853 f_1(z) = 1.801\\ g_1(z) &= 0.999 g_1(0) = 0.999\cdot 1.21 = 1.209\\ g_2(z) &= \frac{m}{m_\mu}\cdot 8g_1(0) = 85.83\,. \end{aligned}\right. \tag{19}$$

([13]) M. Goldberger and S. B. Treiman: *Phys. Rev.*, **110**, 1478 (1958).
([14]) A. Fujii and H. Primakoff: *Nuovo Cimento*, **12**, 327 (1959).

TABLE I. – *The hyperfine and molecular capture rates in units of* s^{-1}. The last row is for Fujii-Primakoff's choice (19).

Case	$g_1(0)$	ϱ_A	g_P/g_A	w_1	w_0	w
1	1.25	0	7.3	13.5	705	623
2	1.25	ϱ_V	6.8	11.8	682	602
3	1.25	$2\varrho_V$	6.4	10.2	661	583
4	1.25	$3\varrho_V$	6.0	8.8	639	563
5	1.20	0	7.5	12.9	661	584
6	1.20	ϱ_V	7.1	11.3	641	566
7	1.20	$2\varrho_V$	6.7	9.9	620	547
8	1.20	$3\varrho_V$	6.3	8.5	599	528
F.P.	1.21	0	8.	14.1	663	589

The experiments on the muon capture in liquid hydrogen are reported recently by three teams [3-5]. The Columbia group gives

$$w_{\text{exp}} = (515 \pm 85)\ \text{s}^{-1}\,,$$

while the combined Chicago-CERN result is

$$w_{\text{xep}} = (409 \pm 62)\ \text{s}^{-1}\,,$$

where a 4% correction due to the admixture of atomic absorption is made to deduce the molecular capture rate. We observed that the theoretical prediction is certainly *higher*, beyond the experimental error.

We can trace back the possible source of discrepancy either in molecular physics or in particle physics, *viz.*

(i) the molecular parameters γ and η are not correctly computed, so that the molecular capture rate is no longer given by eq. (8),

or

(ii) the hyperfine atomic capture rates are not correctly computed,

or else

(iii) errors exist both in the molecular and particle physics part.

WEINBERG [8] remarks that the estimate of γ is subject to the corrections of the order m_μ/m due to the admixture of higher orbitals than $1s\sigma g$, but otherwise the adopted muon wave function is exact. If γ, or η, is overestimated $(15 \sim 20)\%$ we can resolve the difficulty, but we do not have any quantitative judgement at the present time. Supposing that the correct molecular parameters are not too far from the presently adopted value, the situation would become interesting. We assumed

(i) definite G-parity for the currents;

(ii) c.v.c. hypothesis;

(iii) a.c.a.c. hypothesis; and

(iv) the plausible choice for ϱ_A.

In fact the choice of ϱ_A cannot be too arbitrary. We prefer ϱ_A to be $0 \sim \varrho_V$, otherwise the large ϱ_A would imply the dominance of a low mass 1^{+-} state. Thus any of the first three assumptions may be questioned. ADAMS ([7]) once discussed the effect of the possible existence of the second class interactions (which means the assumption (i) is discarded), but it is difficult to draw any definite conclusion because of our absolute lack of knowledge of the form factors of the second class interactions. If the c.v.c. hypothesis or the a.c.a.c. hypothesis, or both, are thrown away, the form factors can be really anything, however the normalization condition $f_1(0) = 1$ and $g_1(0) = (g_A/g_V)_{\beta\ \text{decay}}$ should be kept if the universality is demanded. We construct Tables II, III and IV by adopting an extreme view that « anything » means nought, but still keeping the universality. Namely we repeated the computation with the same particle and molecular parameters, but with the supplementary conditions

$$f_2(z) = 0 \qquad \text{or} \qquad g_2(z) = 0 \qquad \text{or} \qquad f_2(z) = g_2(z) = 0\,,$$

respectively.

TABLE II. – *Capture rates in units of* s^{-1} *without the weak magnetism.*

Case	w_1	w_0	w	Case	w_1	w_0	w
1	6.3	601	530	5	6.0	561	494
2	5.2	580	511	6	5.0	542	477
3	4.2	561	493	7	4.2	523	460
4	3.3	540	475	8	3.4	504	443

TABLE III. – *Capture rates in units of* s^{-1} *without the induced pseudoscalar.*

Case	w_1	w_0	w	Case	w_1	w_0	w
1	6.2	852	749	5	5.0	722	7[illegible]6
2	5.3	814	716	6	4.2	769	676
3	4.5	778	684	7	3.5	735	646
4	3.7	742	652	8	2.9	700	615

TABLE IV. – *Capture rates in units of* s^{-1} *without both the weak magnetism and the induced pseudoscalar.*

Case	w_1	w_0	w	Case	w_1	w_0	w
1	2.5	737	647	5	1.7	692	608
2	1.9	702	616	6	1.2	660	579
3	1.3	669	587	7	0.8	628	552
4	0.9	635	558	8	0.4	597	524

We observe that the exclusion of the weak magnetism decreases the capture rates, the exclusion of the induced pseudoscalar increases the capture rates, and the exclusion of both nearly cancels the opposing effects. It is interesting to see that the present experimental accuracy can distinguish the existence of the weak magnetism or the induced pseudoscalar, provided the molecular parameters are known to a satisfactory accuracy. Hence we appeal the molecular physicists to supply us trustworthy molecular parameters γ and η.

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